



Two scale  
variables

Categorical  
and scale  
variable

Two  
categorical  
variables

# Multivariate descriptives

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- 1 Two scale variables
- 2 Categorical and scale variable
- 3 Two categorical variables

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# Correlation

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Two variables are correlated when the variance in one variable coincides with variance in another variable.

- **Positive correlation:** high values in one variable coincide with high values in another variable.
- **Negative correlation:** high values in one variable coincide with low values in another variable.



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Slightly simplified, for  $X$  to be a cause of  $Y$ , we generally require:

- 1  $X$  to precede  $Y$
- 2  $X$  to correlate with  $Y$  (either positively or negatively)
- 3 no other factor to explain the correlation between  $X$  and  $Y$  (no **confounding factor**)

# Causation: terminology

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If  $X$  causes  $Y$ ,

- $Y$  is called the **dependent variable**, or **outcome variable**, or **response**, or . . . ;
- $X$  is called the **independent variable**, or **explanatory variable**, or **factor**, or . . . .

In political science, most common (unfortunately) is the usage of the terms independent and dependent variables.



# Outline

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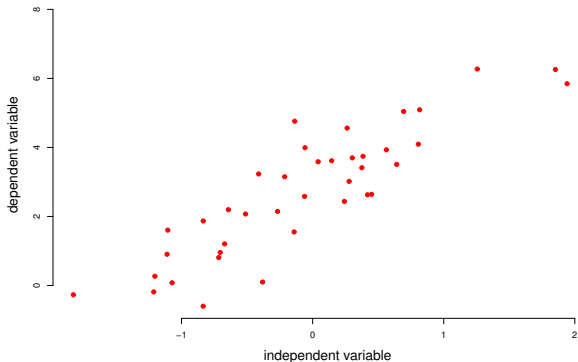
3 Two categorical variables

# Scatter plot

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The relationship between two continuous variables is most easily displayed using a scatter plot.



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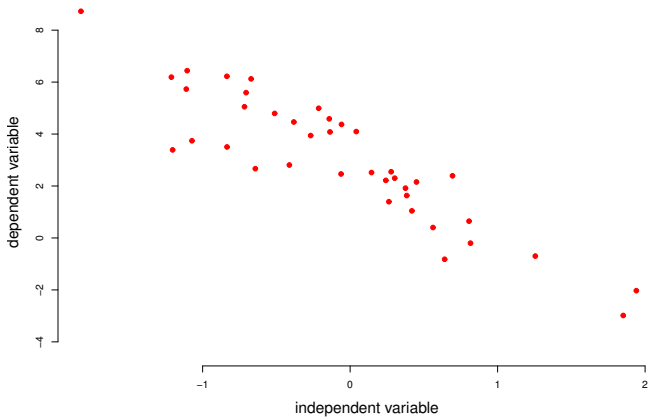
# Scatter plot



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# Covariance

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$$\text{Var}(x) = s^2 = \frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N}$$

$$\text{Cov}(x, y) = \frac{\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{N}$$

- Unbounded - size depends on variance in  $x$  and  $y$
- Positive correlation:  $\text{Cov}(x, y) > 0$
- Negative correlation:  $\text{Cov}(x, y) < 0$

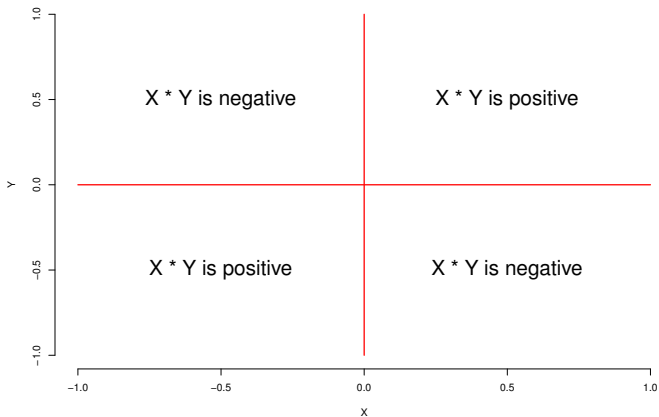
# Covariance: intuition



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# Pearson correlation

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$$r_{xy} = \frac{\text{Cov}(x, y)}{s_x s_y} = \frac{\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^N (x_i - \bar{x})^2} \cdot \sqrt{\sum_{i=1}^N (y_i - \bar{y})^2}}$$

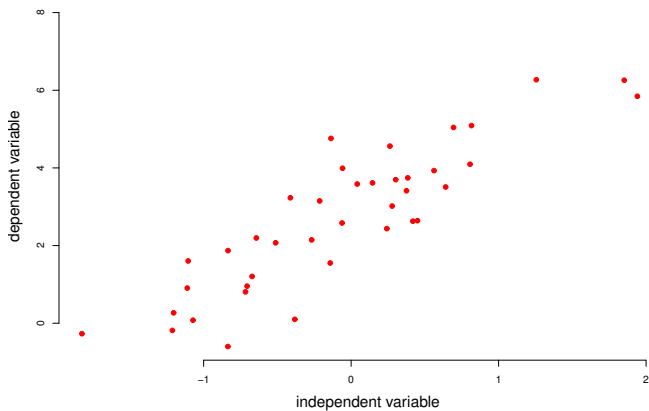
- Bounded between -1 and 1
- Positive correlation:  $r > 0$
- Negative correlation:  $r < 0$



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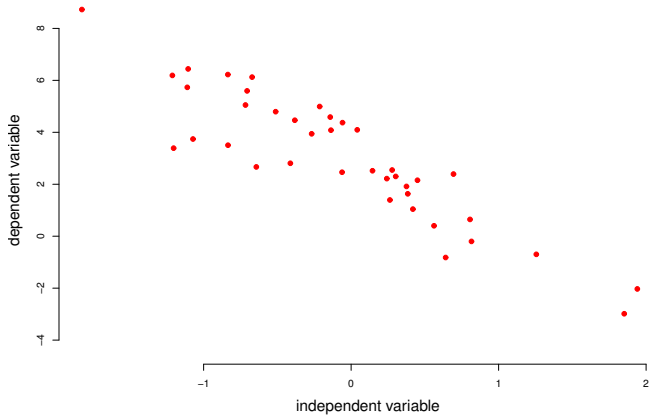
$$\begin{aligned} \text{Var}(x) &= 1.17 & \text{Var}(y) &= 4.95 \\ \text{Cov}(x, y) &= 2.15 & \text{Cor}(x, y) &= 0.89 \end{aligned}$$



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$$\text{Var}(x) = 1.17$$

$$\text{Var}(y) = 10.60$$

$$\text{Cov}(x, y) = -3.35$$

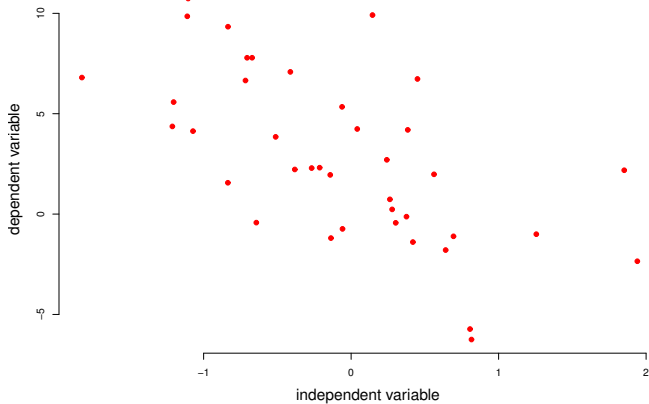
$$\text{Cor}(x, y) = -0.95$$



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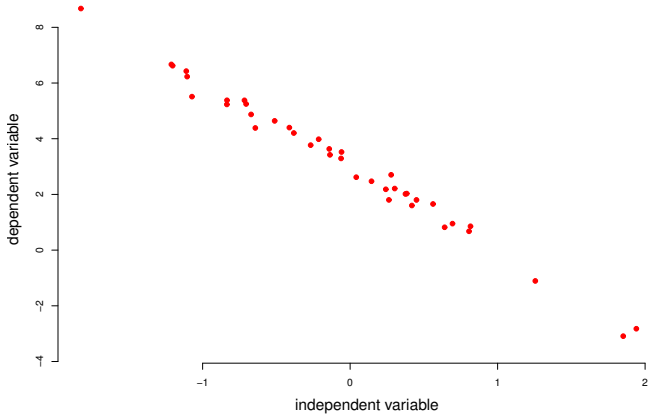
$$\begin{aligned} \text{Var}(x) &= 1.17 & \text{Var}(y) &= 26.80 \\ \text{Cov}(x, y) &= -4.21 & \text{Cor}(x, y) &= -0.75 \end{aligned}$$



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$$\text{Var}(x) = 1.17$$

$$\text{Var}(y) = 10.90$$

$$\text{Cov}(x, y) = -3.58$$

$$\text{Cor}(x, y) = -0.99$$



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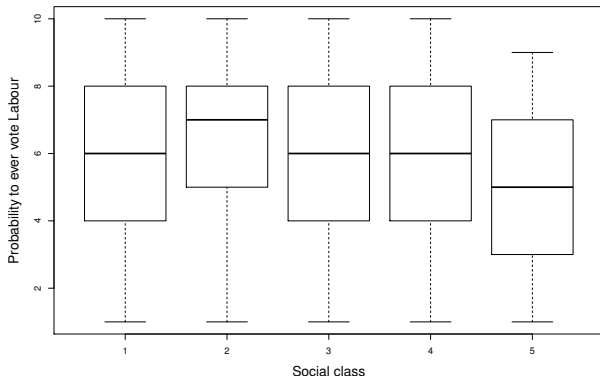
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# Box plots

One way to graphically look at the relation between a categorical and a scale variable is the use of multiple box plots next to each other.



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# Example: English parliament

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Data from a vote in 1844 to reduce the working day for children to 10 hours.

| party         | count | perc. | vote | count | perc. |
|---------------|-------|-------|------|-------|-------|
| liberals      | 150   | 39%   | yes  | 194   | 50.4% |
| conservatives | 235   | 61%   | no   | 191   | 49.6% |
| total         | 385   |       |      | 385   |       |

(<http://www.let.leidenuniv.nl/history/RES/stat/html/les9.html>)

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|               | yes | no  | total |
|---------------|-----|-----|-------|
| liberals      | 94  | 56  | 150   |
| conservatives | 100 | 135 | 235   |
| total         | 194 | 191 | 385   |

Exercise: calculate row and column percentages.

Which are more appropriate?

(<http://www.let.leidenuniv.nl/history/RES/stat/html/les9.html>)