Advanced Quantitative Methods
Lab 7: Maximum Likelihood

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In this question, we will work with the baseline regression model:

$$polity2_i = \beta_0 + \beta_1 energy2_i + \beta_2 ioscore_i + \beta_3 catho80_i.$$

We will extend this model by adding a spatially lagged dependent variable:

$$polity2_i = \beta_0 + \rho W \cdot polity2 + \beta_1 energy2_i + \beta_2 ioscore_i + \beta_3 catho80_i,$$

whereby W is an $n \times n$ matrix identifying neighbouring states and ρ a coefficient to estimate the level of spatial autocorrelation. To estimate the parameters of this spatial autoregressive model, we need the following loglikelihood function (Ward and Gleditsch, 2008, 35):

$$\ell = ln|I - \rho W| - \frac{n}{2} ln(2\pi) - \frac{n}{2} ln(2\sigma^2) - \frac{e'e}{2\sigma^2},$$

where

$$e = (y - \rho W y - X\beta).$$

We will use the following approximation:

$$ln|I - \rho W| = \prod_i (1 - \rho \omega_i),$$

whereby ω_i is the ith eigenvalue of W.[1]

1. Open the `demdev.dta` data set and select cases from 1988 only.
   ```
   library(rio)
   dd <- subset(import("demdev.dta"), year == 1988)
   ```
2. Open the `demdev-W-1988.dta` data set.
3. Run a linear regression to estimate the baseline model.
4. Finish the following loglikelihood function in R for the spatial autoregressive model described above:

```
loglik <- function(theta, y, X, W) {

        s2 <- exp(theta[1])
        rho <- 1 / (1 + exp(-theta[2]))
        beta <- theta[-c(1,2)]

        n <- length(y)
        I <- diag(n)
}
```

[1] In R this will be: `prod(1 - rho * eigen(W)$value)`.

5. Using

```
X <- model.matrix(polity2 ~ energy2 + ioscore + catho80, data = dd)
y <- dd$polity2
W <- as.matrix(import("demdev-W-1988.dta"))
```

to generate the data matrices, estimate the spatial regression model using `optim()`.

6. Using the output from `optim()`, calculate the standard errors for the β-coefficients.
7. Perform t-tests for the β-coefficients, reporting the p-values.
8. Calculate the estimated values of ρ and σ^2.

Ward, Michael D and Kristian Skrede Gleditsch. 2008. *Spatial regression models*. Vol. 155 Sage.