

Advanced Quantitative Methods

Lab 1: Math (p)review

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1.

$$\mathbf{A} = \begin{bmatrix} 2 & 5 \\ 8 & 3 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

Calculate

- (a) $\mathbf{A} + \mathbf{B}$
- (b) $\mathbf{A} - \mathbf{B}$
- (c) \mathbf{AB}
- (d) \mathbf{AB}'

2.

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 2 \\ 3 & 4 & 6 \end{bmatrix}$$

Calculate

- (a) $\frac{1}{2}\mathbf{A}$
- (b) $\mathbf{A} + \mathbf{A}$
- (c) \mathbf{AA}'
- (d) $\mathbf{A}'\mathbf{A}$
- (e) $\text{tr}(\mathbf{A}'\mathbf{A})$

3.

$$\mathbf{X} = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 3 \\ 1 & -1 & 3 \\ 1 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} 4 \\ 6 \\ 5 \\ 6 \\ 4 \end{bmatrix} \quad \hat{\boldsymbol{\beta}} = \begin{bmatrix} 1.3 \\ 0.4 \\ 1.5 \end{bmatrix}$$

Calculate

- (a) $\mathbf{X}\hat{\boldsymbol{\beta}}$
- (b) $\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{e}$
- (c) $\mathbf{e}'\mathbf{e}$

4.

$$f(x) = 3 + 4x - 3x^2$$

- (a) Find for which x we have $f(x) = 0$.
- (b) Calculate the derivative of $f(x)$.
- (c) Find the slope for $x = 1$.

- (d) Find for which x we have the derivative $f'(x) = 0$.
5. Find the derivative towards x of (taken from ?)
- (a) $y = 4x - 3$
 - (b) $y = 1/x^2$
 - (c) $y = (2x - 1)(2x + 1)$
6. (a) Find the derivative towards β of $\mathbf{X}\beta$
 (b) Find the derivative towards \mathbf{x} of $\mathbf{x}'\mathbf{W}'\mathbf{W}\mathbf{x}$
 (c) Find the derivative towards β of $\mathbf{X}\beta + \beta'\mathbf{X}'\mathbf{X}\beta$
7. Let x be a random variable containing only ones and zeros. Let π be the probability of observing a one. Show that
- (a) $E(x) = \pi$
 - (b) $var(x) = \pi(1 - \pi)$

Ayres, Frank and Elliot Mendelson. 1999. *Schaum's outline of calculus*. 4th ed. McGraw-Hill.