

# Advanced Quantitative Methods

## Lab 8: Maximum Likelihood

Johan A. Elkink  
jos.elkink@ucd.ie

6 April 2018

In this question, we will work with the baseline regression model:

$$polity2_i = \beta_0 + \beta_1 energy2_i + \beta_2 ioscore_i + \beta_3 catho80_i.$$

We will extend this model by adding a spatially lagged dependent variable:

$$polity2_i = \beta_0 + \rho W \cdot polity2 + \beta_1 energy2_i + \beta_2 ioscore_i + \beta_3 catho80_i,$$

whereby  $W$  is an  $n \times n$  matrix identifying neighbouring states and  $\rho$  a coefficient to estimate the level of spatial autocorrelation. To estimate the parameters of this spatial autoregressive model, we need the following loglikelihood function (Ward and Gleditsch, 2008, 35):

$$\ell = \ln|I - \rho W| - \frac{n}{2} \ln(2\pi) - \frac{n}{2} \ln(2\sigma^2) - \frac{e'e}{2\sigma^2},$$

where

$$e = (y - \rho W y - X\beta).$$

We will use the following approximation:

$$\ln|I - \rho W| = \prod_i (1 - \rho \omega_i),$$

whereby  $\omega_i$  is the  $i$ th eigenvalue of  $W$ .<sup>1</sup>

1. Open the demdev.dta data set and select cases from 1988 only.  
`library(rio)`  
`dd <- subset(import("demdev.dta"), year == 1988)`
2. Open the demdev-W-1988.dta data set.
3. Run a linear regression to estimate the baseline model.
4. Finish the following loglikelihood function in R for the spatial autoregressive model described above:

---

```
loglik <- function(theta, y, X, W) {  
  
  s2 <- exp(theta[1])  
  rho <- 1 / (1 + exp(-theta[2]))  
  beta <- theta[-c(1,2)]  
  
  n <- length(y)  
  I <- diag(n)  
  
}
```

---

<sup>1</sup>In R this will be: `prod(1 - rho * eigen(W)$value)`.

## 5. Using

---

```
X <- model.matrix(polity2 ~ energy2 + ioscore + catho80, data = dd)
y <- dd$polity2
W <- as.matrix(import("demdev-W-1988.dta"))
```

---

to generate the data matrices, estimate the spatial regression model using `optim()`.

6. Using the output from `optim()`, calculate the standard errors for the  $\beta$ -coefficients.
7. Perform  $t$ -tests for the  $\beta$ -coefficients, reporting the  $p$ -values.
8. Calculate the estimated values of  $\rho$  and  $\sigma^2$ .

Ward, Michael D. and Kristian Skrede Gleditsch. 2008. *Spatial regression models*. Vol. 155 Sage.