Advanced Quantitative Methods
Homework 1: Math review

Johan A. Elkink
jos.elkink@ucd.ie

Due January 26, 2011, 9am

Please submit by email in PDF format

1. Consider the following three matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 7 & 8 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 5 & 6 \\ 1 & 6 \\ 4 & 7 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

   Write down the result of

   (a) (5%) $\mathbf{A} + \mathbf{B}'$

   (b) (5%) $\frac{1}{2}\mathbf{AB}$

   (c) (5%) $\mathbf{ACA}'$

2. (8%) Write down the following matrix: $\mathbf{A} = [a_{ij}]_{4\times 3}$, where $a_{ij} = 2i + 3(j - \frac{1}{2})$.

3. (8%) For matrix

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \end{bmatrix},$$

   write down what $\mathbf{X}'\mathbf{X}$ looks like.

4. In OLS, we estimate β with $\hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$. Getting the predicted values of $\mathbf{y}$ thus becomes: $\hat{\mathbf{y}} = \mathbf{X}\hat{\beta} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} = \mathbf{My}$. Show that $\mathbf{M}$ is both idempotent (10%) and symmetric (10%).

5. Present the derivatives $\frac{dy}{dx}$ for the following functions:

   (a) (5%) $y = \frac{1}{2}x^2 + \frac{1}{3}x^3 + 1$

   (b) (5%) $y = 2x^{-1} + 3x$

   (c) (5%) $y = (3x^2 + x)^2$

   (d) (5%) $y = \frac{3x^2}{3x} + 2x$

6. Suppose $P(A) = \frac{2}{9}$ and $P(B) = \frac{7}{9}$ are independent events, what are

(a) (3%) $P(A|B)$

(b) (3%) $P(A \cap B)$

(c) (3%) $P(A \cup B)$

7. (10%) Suppose a medical test provides a false positive or false negative in 1% of the tests. The disease is rare and occurs for only one in a thousand in the overall population. The test is taken for all citizens at a particular age and does not depend on observed symptoms or particularly risky behaviour by the patient. What is the probability that a patient has, in fact, the disease, when the test results in a diagnosis that the patient does? You should be using Bayes' theorem:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\neg A)P(\neg A)}$$

8. Let x be a random variable containing only ones and zeros. Let π be the probability of observing a one. Show that

(a) (5%) $E(x) = \pi$

(b) (5%) $var(x) = \pi(1 - \pi)$