t-tests and F-tests in regression

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5 April 2012

Outline

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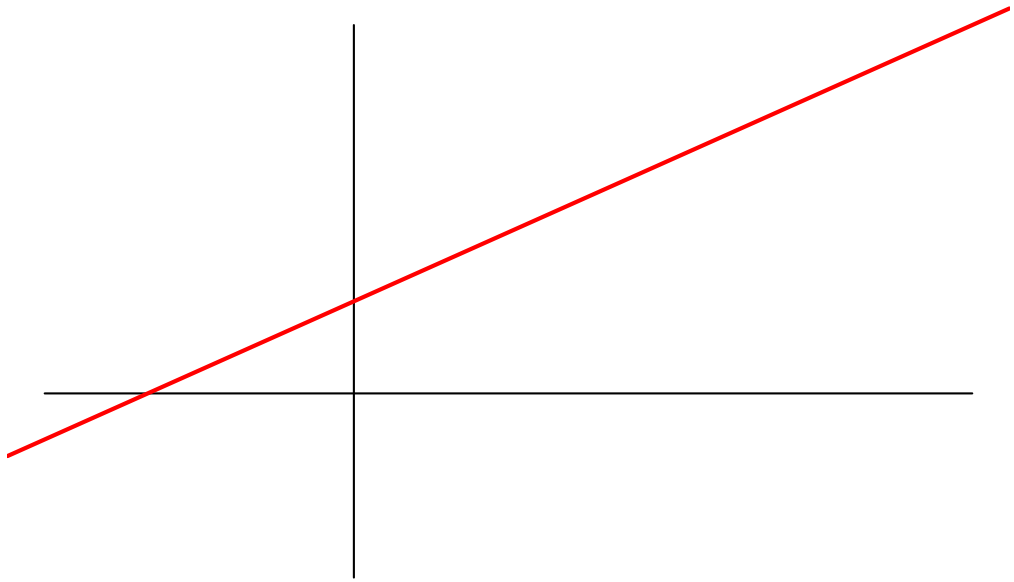
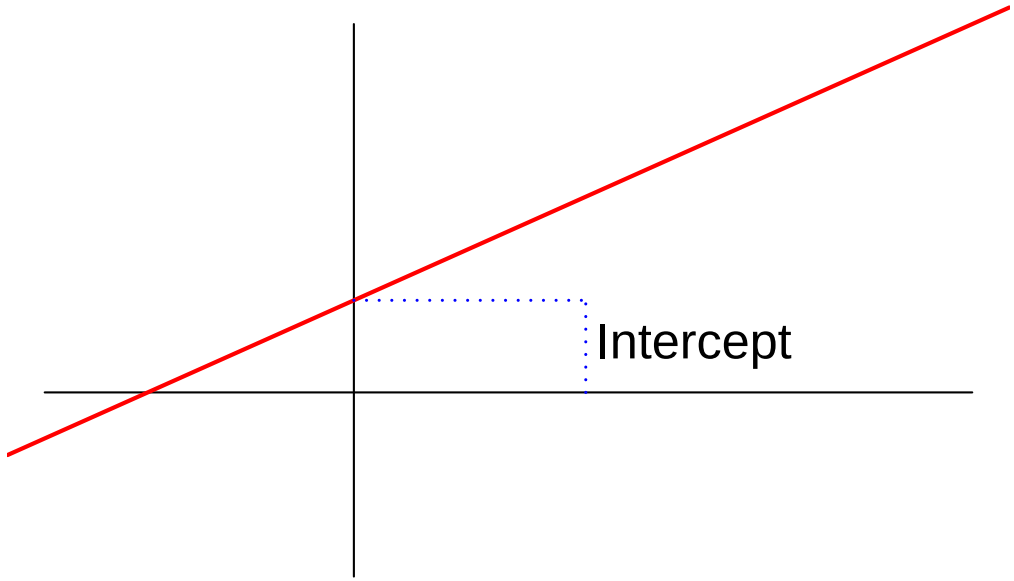
Linear equations



$$y = Intercept + Slope * x$$

Linear equations



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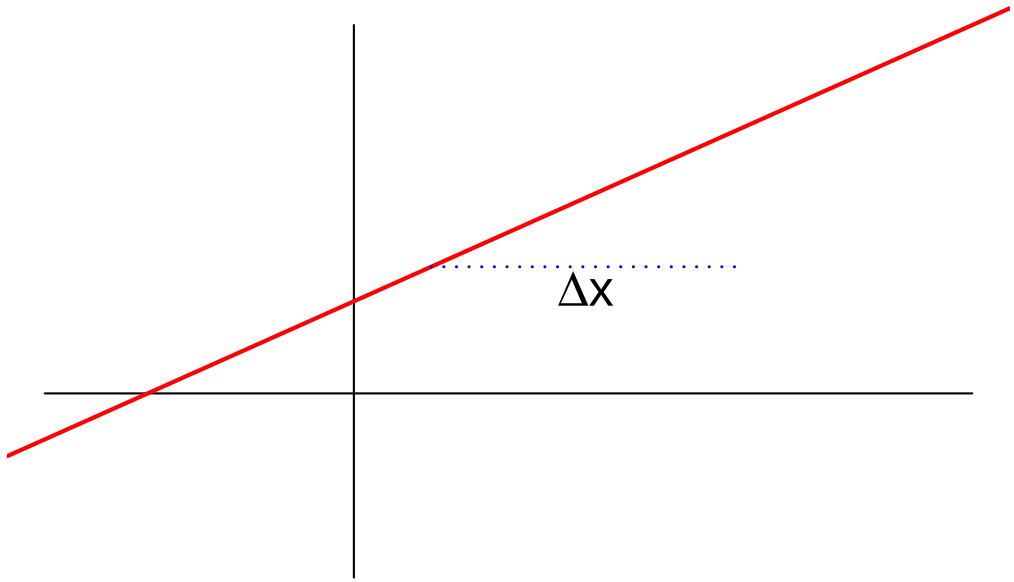

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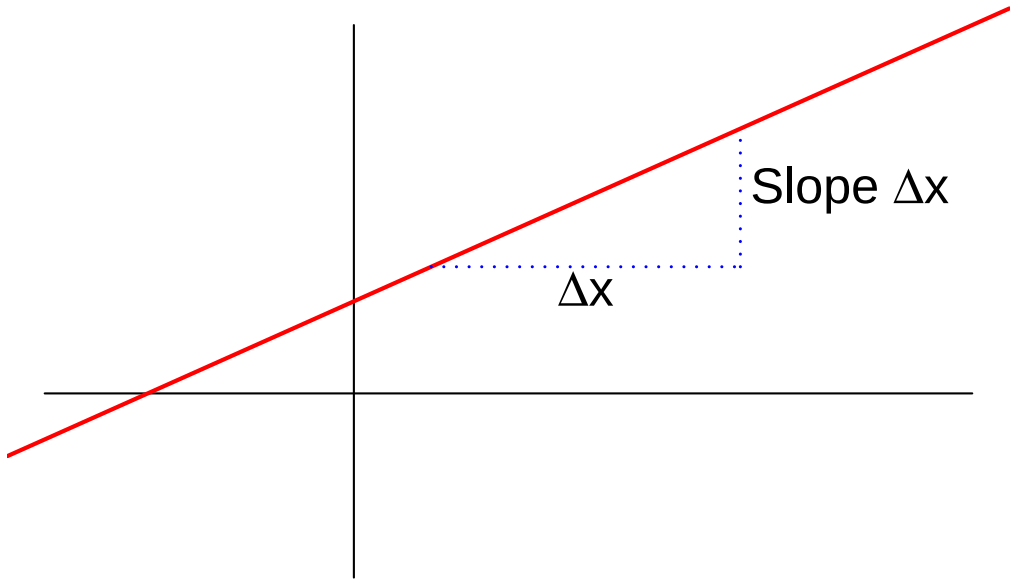

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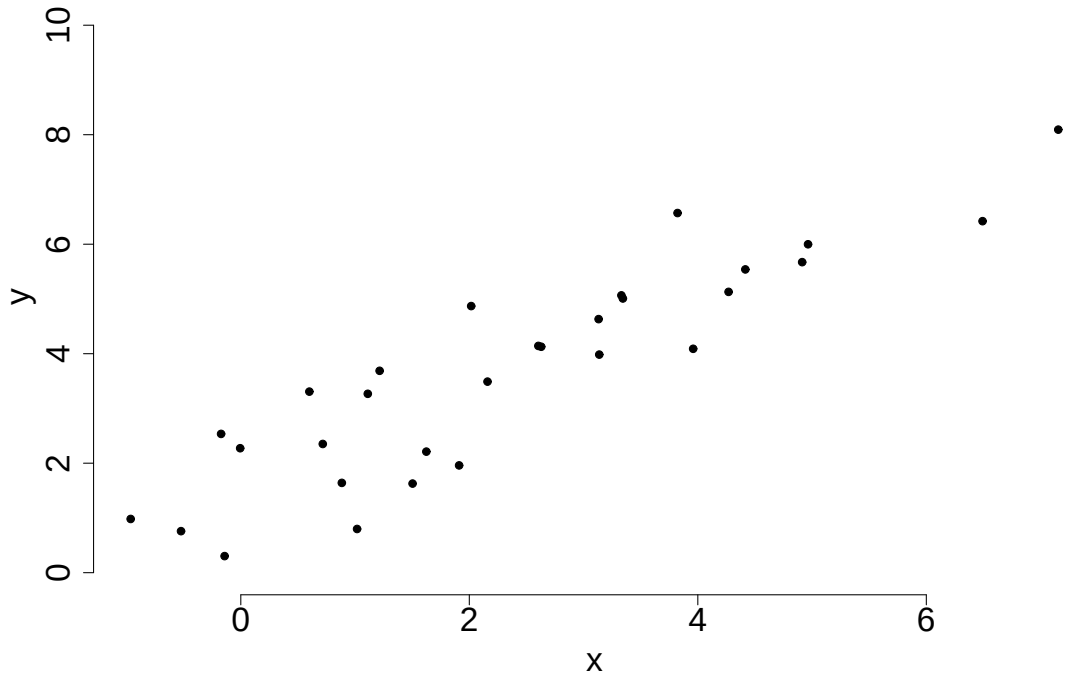
Simple regression model



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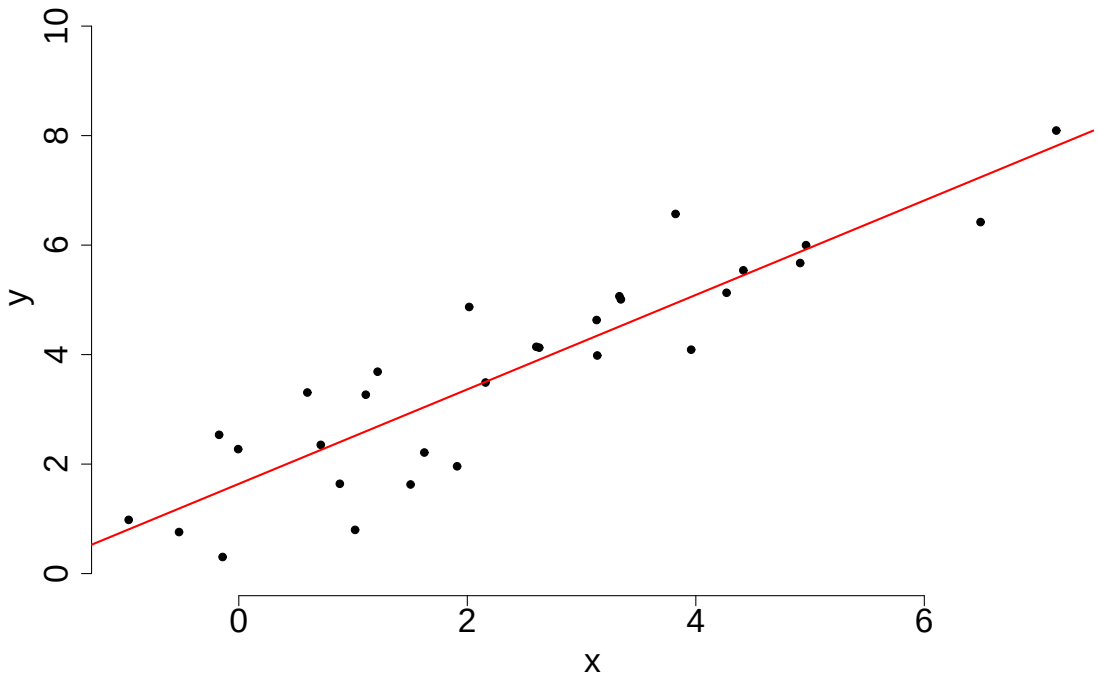

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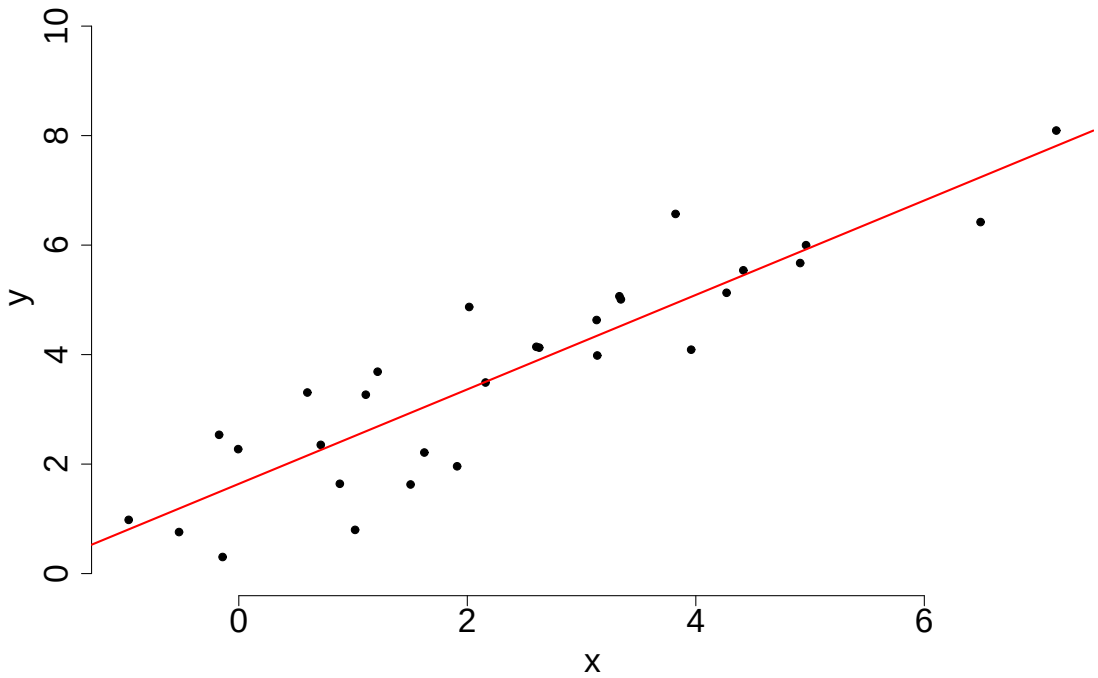

$$y = \beta_0 + \beta_1 x$$

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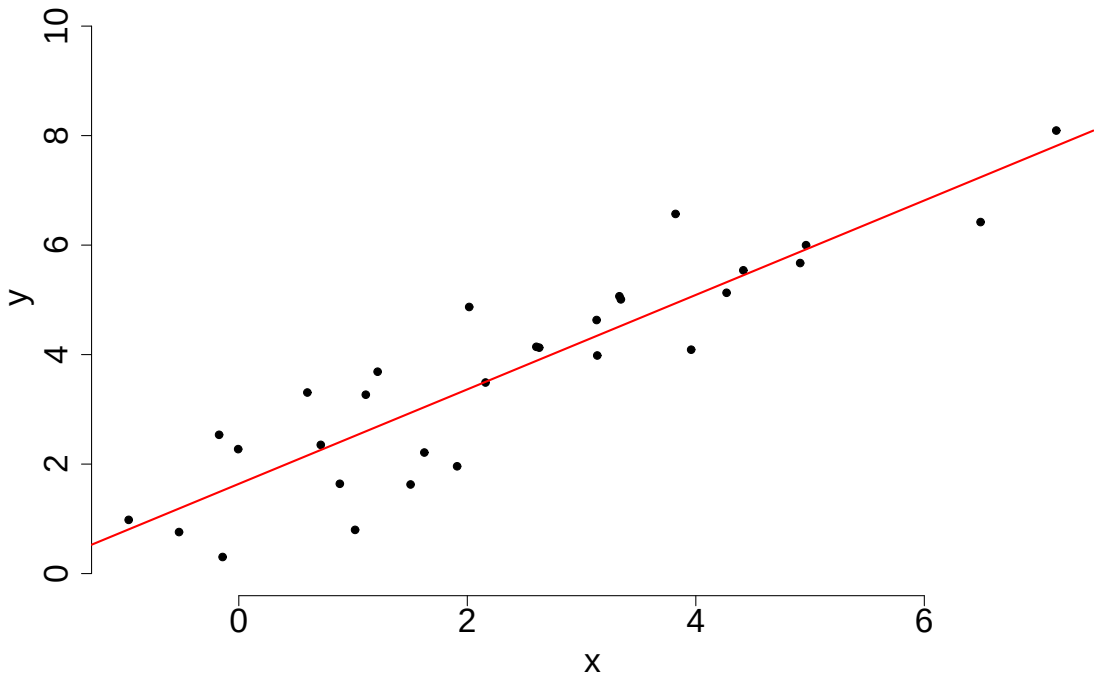

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

Notation

y_i	Value on the dependent variable for case i
x_i	Value on the independent variable for case i
$\bar{x}$	Mean value on the independent variable for case i
ε_i	The error for case i: $\varepsilon_i = y_i - \hat{y}_i$
β_k	True coefficient for variable k
$\hat{\beta}_k$	Estimated coefficient for variable k
$\hat{y}_i$	Predicted value on the dependent variable for case i

Ordinary Least Squares

"Quickly put, the regression line is chosen to minimize the RSS; it has slope $\hat{\beta}_1$, intercept $\hat{\beta}_0$, and goes through the point $(\bar{x}, \bar{y})$. Furthermore, the estimate for σ^2 is $\hat{\sigma}^2 = RSS/(n-2)$" (Verzani 2005: 280).

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Breakdown of variance

Total Sum of Squares (TSS): $\sum_{i=1}^{N}(y_i - \bar{y})^2$

Explained Sum of Squares (ESS): $\sum_{i=1}^{N}(\hat{y}_i - \bar{y})^2$

Residual Sum of Squares (RSS): $\sum_{i=1}^{N}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{N}\varepsilon_i^2$

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Sometimes the second is called "regression sum of squares" (RSS) and the third "errors sum of squares" (ESS), which might in fact be more accurate, since ε really represents errors, not residuals, in this specification. Beware the confusion!

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For simple linear regression (i.e. one independent variable), R^2 is the same as the correlation coefficient, Pearson's r, squared.

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Or perform a hypothesis test along the lines of:

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$$H_1 : \beta_1 \neq 0$$

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For multiple regression, this would generalize to:

$$F = \frac{ESS/(k-1)}{RSS/(n-k)} \sim F_{k-1,n-k}$$

Outline

Exercise

Open `oecd_1960.sav`.

Repeat for both industry (IND) and services (AGR):

1. Regress percentage in sector on income per capita.
2. Interpret the regression results.
3. Evaluate the model fit.
4. Interpret the t- and F-tests.

Exercise

Open `bes_class_data.sav` and investigate:

- whether left-right self-placement (`lr`) and explains trust in politics (`trustpol`)
- whether left-right self-placement influences attitude towards EU membership (`eumember`)