Advanced Quantitative Methods
Homework 1

Johan A. Elkink
jos.elkink@ucd.ie

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Please submit by email in PDF format. Add R code in a separate `.R` *file, or SPSS code in a separate* `.sps` *file, or Stata code in a separate* `.do` *file, or the code for any other package you use separately. Note that if you do not use Latex, there is an "Equation" entry in Microsoft Word under "Insert" that will allow you to include matrices and other mathematical equations.*

1. Consider the following vector $\mathbf{e}$ and matrix $\mathbf{W}$:

$$\mathbf{e} = \begin{bmatrix} 0.16 \\ -1.21 \\ 0.15 \\ 0.90 \end{bmatrix} \qquad \mathbf{W} = \begin{bmatrix} 0.30 & 0 & 0 & 0 \\ 0 & 0.06 & 0 & 0 \\ 0 & 0 & 0.48 & 0 \\ 0 & 0 & 0 & 0.15 \end{bmatrix}$$

   Write down the result of

   (a) (5%) $\sum_{i=1}^{4} e_i$ and $\sum_{i=1}^{4} w_{ii}$
   (b) (5%) $\mathbf{e}'\mathbf{e}$
   (c) (5%) $\mathbf{e}'\mathbf{W}\mathbf{e}$
   (d) (5%) $\mathbf{e}\mathbf{e}'$

2. Consider the following matrices:

$$\mathbf{X} = \begin{bmatrix} 1 & -1.0 & -0.5 \\ 1 & -1.7 & -0.3 \\ 1 & -0.4 & 1.1 \\ 1 & -2.2 & 0.2 \\ 1 & -0.5 & 0.1 \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} 1.9 \\ 0.9 \\ 0.8 \\ -0.3 \\ 1.2 \end{bmatrix} \qquad \beta = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \qquad \hat{\beta} = \begin{bmatrix} 2.21 \\ 1.04 \\ -0.99 \end{bmatrix}$$

   Write down the result of

   (a) (5%) $\mathbf{X}\beta$
   (b) (5%) $\mathbf{y} - \mathbf{X}\beta$
   (c) (5%) $\varepsilon'\varepsilon$, where $\varepsilon = \mathbf{y} - \mathbf{X}\beta$
   (d) (5%) $\mathbf{e}'\mathbf{e}$, where $\mathbf{e} = \mathbf{y} - \mathbf{X}\hat{\beta}$

(e) (5%) $\mathbf{X}'\mathbf{X}$

3. When we run an OLS regression, we estimate the coefficients using $\hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$. We can subsequently calculate the residuals as

$$\mathbf{e} = \mathbf{y} - \hat{\mathbf{y}} = \mathbf{y} - \mathbf{X}\hat{\beta} = \mathbf{y} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} = (\mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}')\mathbf{y} = \mathbf{M}\mathbf{y}.$$

   (a) (5%) Show that $\mathbf{M}$ is symmetric ($\mathbf{M}' = \mathbf{M}$).

   (b) (5%) Show that $\mathbf{M}$ is idempotent ($\mathbf{M}'\mathbf{M} = \mathbf{M}$).

4. This exercise is meant as a revision of multiple regression. You will need the `asiabaro.dta` data set, which contains data from the AsiaBarometer. A separate file (`asiabaro.txt`) describes the variables in the data.

   Make sure you look at the distribution of each variable such that no incorrect data are included and make corrections where necessary.

   Investigate the following regression:

$$evaldemoc_i = \beta_0 + \beta_1 suitdemoc_i + \beta_2 prefdemoc_i + \beta_3 trustgovt_i + \beta_4 urban_i + \beta_5 female_i + \varepsilon_i,$$

   whereby the model is estimated for the entire data set, and for each of the three countries separately.

   (a) (15%) Present a regression table properly formatted as for a publication, with a column for each model.

   (b) (10%) Plot, based on the regression for all countries together, `evaldemoc` as a function of `trustgovt`, including the estimated regression line. Assume all other variables at their median (ordinal or scale variables) or mode (nominal variables) value.

   (c) (20%) Write a one page report in which you fully interpret the regression output (coefficients, standard errors, R^2). What does it tell you about the evaluation of the current level of democracy? Focus in particular on the extent to which trust in the current government might explain the evaluation of the current government as a democracy.